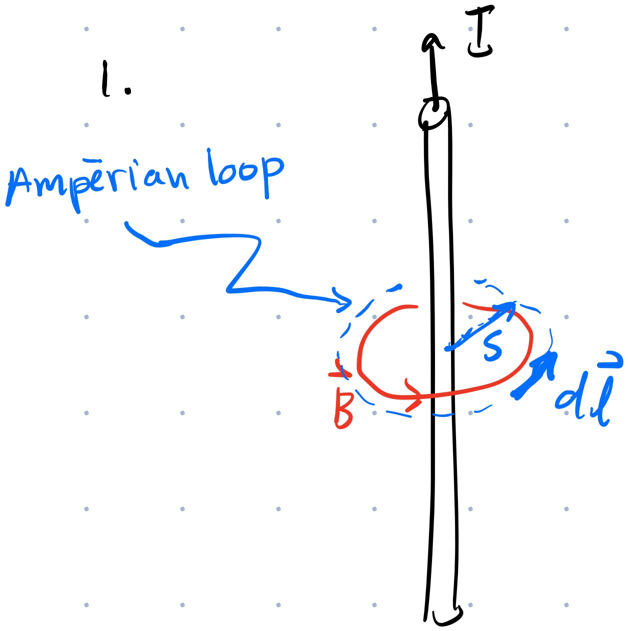
PHYS 301 - Tutorial #7 Sol'ns - Nov. 4, 2024

1.



Know I creates $\vec{B}$ that loops around current.

Pick an Ampèrian loop (i.e. integration path) that matches the symmetry of $\vec{B}$

Ampère's Law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$I_{encl} = I$ (current through our loop).

$\vec{B} \cdot d\vec{l} = B\,dl$ since $\vec{B} \parallel d\vec{l}$ everywhere on loop.

Expect B is const. in magnitude everywhere on loop since distance from loop to current is always s.

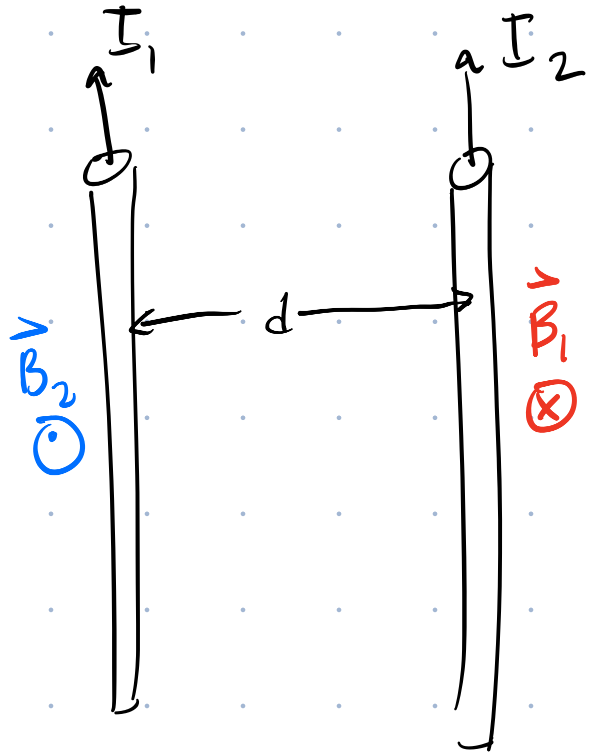
$$\therefore \oint \vec{B} \cdot d\vec{l} = B \int dl = B \underbrace{2\pi s}_{\text{loop circumference.}}$$

$\therefore B\, 2\pi s = \mu_0 I \implies B = \dfrac{\mu_0 I}{2\pi s}$

by RHR dir'n of $\vec{B}$ is $\hat{\phi}$

$$\boxed{\therefore \vec{B} = \frac{\mu_0 I}{2\pi s}\,\hat{\phi}}$$, as expected.

2.



I_1 creates magnetic field $\vec{B}_1$ at position of I_2.

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \quad \text{into screen @ } I_2$$

Likewise, I_2 creates $\vec{B}_2$ @ I_1

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \quad \text{out of screen @ } I_1$$

Force on I_2 due to $\vec{B}_1$ is:

$$\vec{F}_{1 \text{ on } 2} = \int (\vec{I}_2 \times \vec{B}_1)\, d\ell$$

by RHR $\vec{I}_2 \times \vec{B}_1$ is to left (towards I_1)

Since $\vec{I}_2 \perp \vec{B}_1$, $|\vec{I}_2 \times \vec{B}_1| = I_2 B_1$

If the currents are steady, then $\vec{B}$'s are const.

$$\therefore F_{1on2} = I_2 B_1 \int dl = I_2 B_1 l$$

$$\therefore \underbrace{f_{1on2}}_{\text{force per unit length}} = \frac{F_{1on2}}{l} = I_2 B_1 = I_2 \frac{\mu_0 I_1}{2\pi d}$$

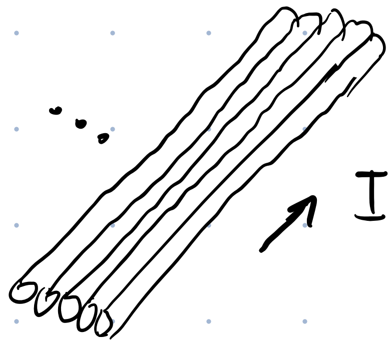
$$\boxed{\therefore f_{1on2} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$

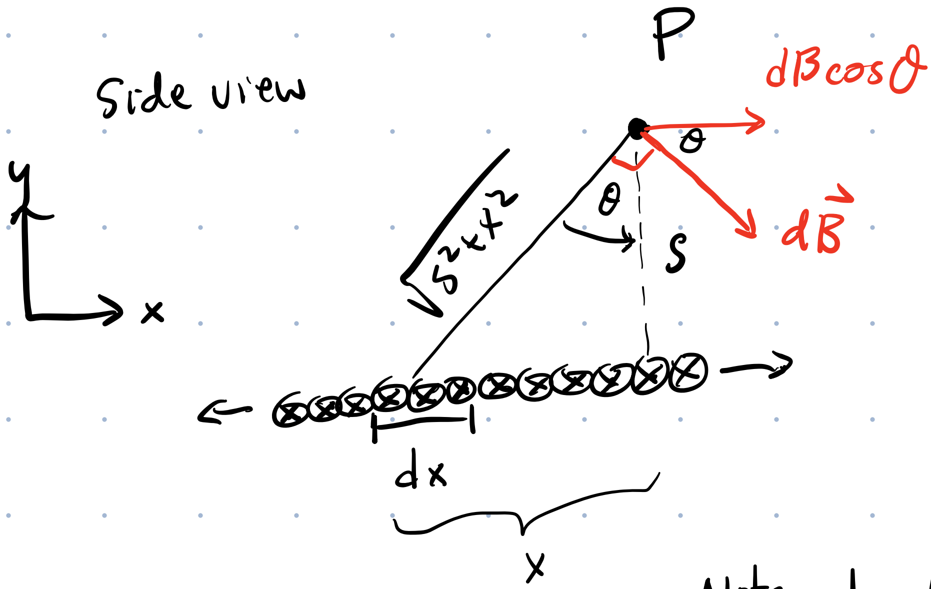
Likewise, find $f_{2on1} = f_{1on2}$, but in opp. dir'n.

If $I_1 \parallel I_2$, force is attractive.

If I_1 & I_2 are antiparallel, then magnitude of the force is unchanged. It is just repulsive.

3.





$$\cos\theta = \frac{s}{\sqrt{s^2+x^2}}$$

only the horizontal component of $d\vec{B}$ will survive.

Note also that

$$\tan\theta = \frac{x}{s}$$

$$dI = K\,dx$$

$$dB = \frac{\mu_0\, dI}{2\pi\sqrt{s^2+x^2}} = \frac{\mu_0 K\, dx}{2\pi\sqrt{s^2+x^2}}$$

Parallel component

$$dB_{\parallel} = dB\cos\theta = \frac{\mu_0 K s\, dx}{2\pi(s^2+x^2)}$$

$$\therefore B_{\parallel} = \int dB_{\parallel} = \int_{x=-\infty}^{\infty} \frac{\mu_0 K s\, dx}{2\pi(s^2+x^2)}$$

Make substitution $x = s\tan\theta$

$x = -\infty,\ \theta = -\pi/2$

$x = +\infty,\ \theta = +\pi/2$

$$dx = s\sec^2\theta\, d\theta$$

$$s^2 + x^2 = s^2(1 + \tan^2\theta) = s^2\sec^2\theta$$

$$\therefore B_{\parallel} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\mu_0 K \cancel{s^2} \cancel{\sec^2\theta}\, d\theta}{2\pi \cancel{s^2} \cancel{\sec^2\theta}}$$

$$= \frac{\mu_0 K}{2\pi} \int_{\theta=-\pi/2}^{\pi/2} d\theta = \frac{\mu_0 K}{2}$$

$\therefore$ above sheet of current

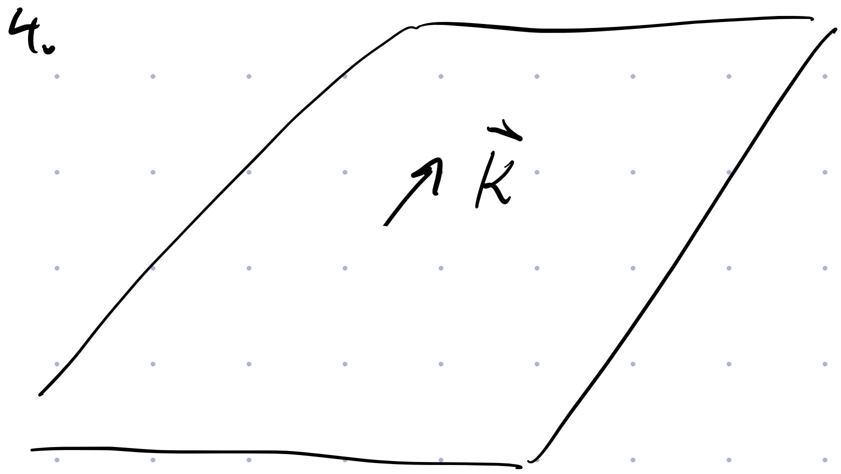
$$\boxed{\vec{B} = \frac{\mu_0 K}{2}\hat{x}}$$

below sheet of current

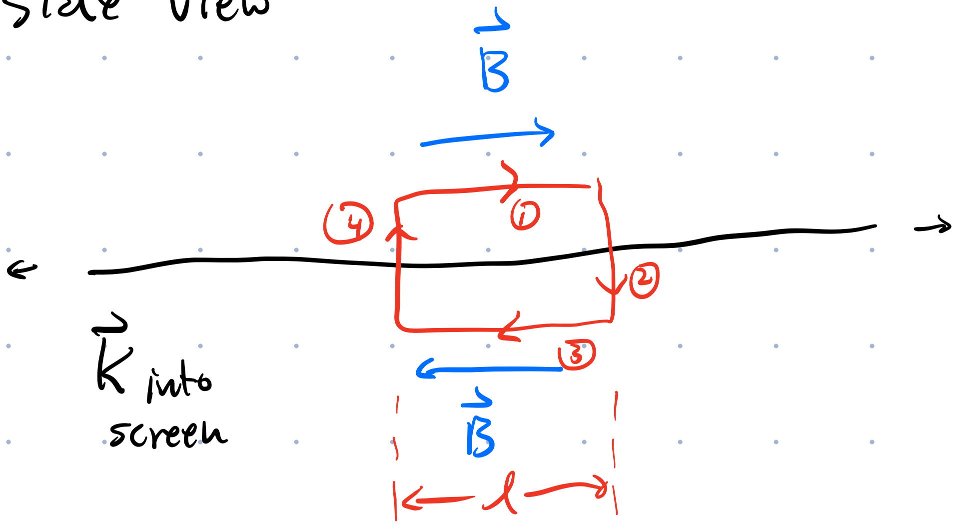
$$\boxed{\vec{B} = -\frac{\mu_0 K}{2}\hat{x}}$$

Magnitude of $\vec{B}$ similar in form to $\vec{E}$ due to a sheet of charge $E = \frac{\sigma}{2\varepsilon_0}$

4.



side view



- Pick an Amperian loop. (red loop).
- $I_{encl} = K\ell$
- $\oint \vec{B}\cdot d\vec{\ell} = \underbrace{\int_{1} \vec{B}\cdot d\vec{\ell}}_{B\,d\ell} + \underbrace{\cancelto{0}{\int_{2} \vec{B}\cdot d\vec{\ell}}}_{\vec{B}\perp d\vec{\ell}} + \underbrace{\int_{3} \vec{B}\cdot d\vec{\ell}}_{B\,d\ell} + \underbrace{\cancelto{0}{\int_{4} \vec{B}\cdot d\vec{\ell}}}_{\vec{B}\perp d\vec{\ell}}$

For ① & ②, $\vec{B}$ always same distance from K, $\therefore$ B is const.

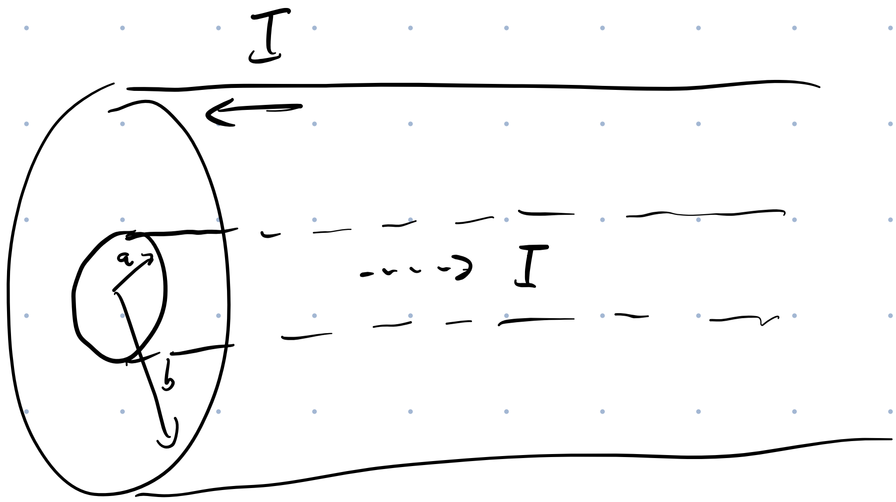
$$\therefore \quad B\int_{①} dl + B\int_{②} dl = \mu_0 K l$$

$$\therefore \quad 2B\cancel{l} = \mu_0 K \cancel{l} \Rightarrow B = \frac{\mu_0 K}{2}$$

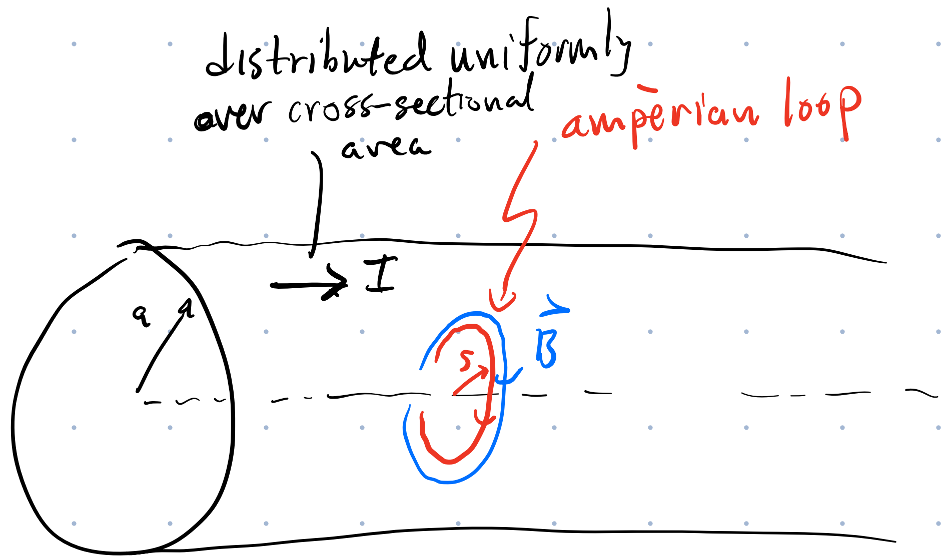
$$\boxed{\vec{B}_{above} = \frac{\mu_0 K}{2}\hat{x} \qquad \vec{B}_{below} = -\frac{\mu_0 K}{2}\hat{x}}$$

same as before!

5.



(a) $s < a$



$$I_{encl} = I\,\frac{\cancel{\pi} s^2}{\cancel{\pi} a^2} = I\left(\frac{s}{a}\right)^2$$

Expect $\vec{B}$ to loop around central axis by RHR.

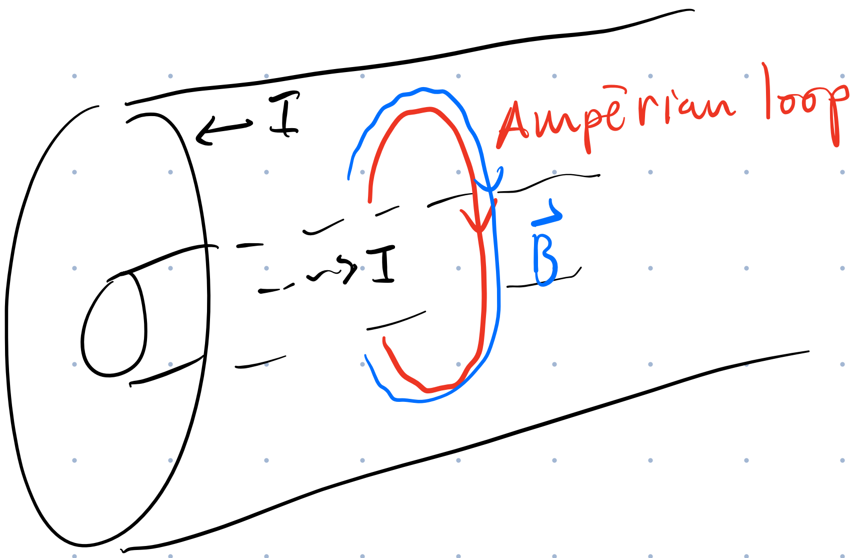
$\therefore\ \vec{B}\cdot d\vec{l} = B\,dl$ & B is const.

$$\therefore \oint \vec{B}\cdot d\vec{l} = B\oint dl = B\,2\pi s$$

$$\therefore B\,2\pi s = \mu_0 I\left(\frac{s}{a}\right)^2$$

$$\boxed{\therefore \vec{B} = \frac{\mu_0 I s}{2\pi a^2}\,\hat{\phi}} \qquad s < a.$$

(b)



Like in (a) $\oint \vec{B}\cdot d\vec{l} = B\,2\pi s$

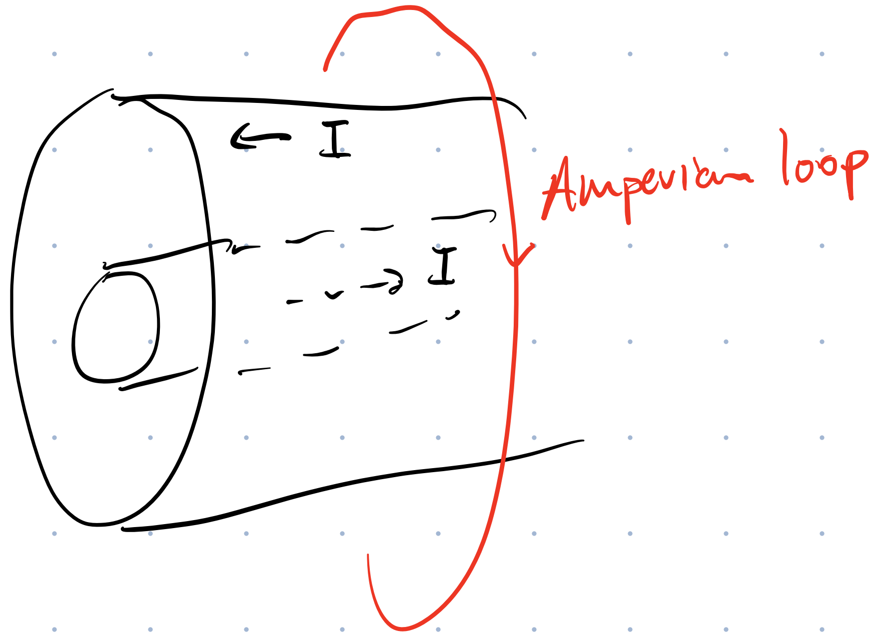
However, now $I_{encl} = I$ (entire current of centre conductor)

$$\therefore B\,2\pi s = \mu_0 I$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}} \qquad a < s < b$$

Same as if we had a line of current I concentrated at the central axis.

(c)



This time, $I_{encl} = +I - I = 0$

centre conductor

outer conductor

$$\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 \cancel{I_{encl}}^{\,0} = 0$$

$$\boxed{\therefore\ B = 0 \text{ is the only possibility.}}$$